

Section 5.10, additional algebra

From [5.94] $B = \frac{A \sin kx'}{\sin k(x' - L)}$, so substituting back into [5.93] gives

$$kA \cos kx' + 1 = \frac{kA \cos k(x' - L) \sin kx'}{\sin k(x' - L)}.$$

So

$$\begin{aligned} 1 &= kA \left[\frac{\cos k(x' - L) \sin kx'}{\sin k(x' - L)} - \cos kx' \right] = kA \left[\frac{\cos k(x' - L) \sin kx' - \sin k(x' - L) \cos kx'}{\sin k(x' - L)} \right], \\ &= kA \left[\frac{(\cos kx' \cos kL + \sin kx' \sin kL) \sin kx' - (\sin kx' \cos kL - \cos kx' \sin kL) \cos kx'}{\sin k(x' - L)} \right], \\ &= kA \left[\frac{\sin kL (\sin^2 kx' + \cos^2 kx') + \cos kL (\cos kx' \sin kx' - \sin kx' \cos kx')}{\sin k(x' - L)} \right], \\ &= kA \left[\frac{\sin kL}{\sin k(x' - L)} \right]. \end{aligned}$$

Thus

$$A(x') = \frac{\sin k(x' - L)}{k \sin kL} \quad \text{and} \quad B(x') = \frac{\sin kx'}{k \sin kL}.$$