The low order spherical harmonics Y_l^m can be re-written as:

$$Y_0^0 \propto \text{constant},$$

$$Y_1^0 \propto \cos\theta \propto \frac{z}{r},$$

$$Y_1^{\pm 1} \propto \mp \sin\theta \, \mathrm{e}^{\pm \mathrm{i}\phi} \propto \mp \frac{(x \pm \mathrm{i}y)}{r},$$

$$Y_2^0 \propto 3\cos^2\theta - 1 \propto \frac{3z^2 - r^2}{r^2},$$

$$Y_2^{\pm 1} \propto \mp \sin\theta \cos\theta \, \mathrm{e}^{\pm \mathrm{i}\phi} \propto \mp \frac{(x \pm \mathrm{i}y)z}{r^2},$$

$$Y_2^{\pm 2} \propto \sin^2\theta \, \mathrm{e}^{\pm \mathrm{i}2\phi} \propto \frac{(x \pm \mathrm{i}y)^2}{r^2}.$$

For a particular l value, there are linear combinations of these which produce *real* functions that correspond to the s, p, d... orbitals used in chemistry:

$$s(l=0): \ \propto \text{constant};$$

$$p(l=1): \ \propto \frac{x}{r}, \ \frac{y}{r}, \ \frac{z}{r};$$

$$d(l=2): \ \propto \frac{xy}{r^2}, \ \frac{xz}{r^2}, \ \frac{yz}{r^2}, \ \frac{3z^2 - r^2}{r^2}, \ \frac{x^2 - y^2}{r^2}.$$

(These are sometimes called ‘cubic harmonics’).